# Constraints on color dipole-nucleon cross section from diffractive heavy quarkonium production

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We study the hard color dipole-nucleon cross section within perturbative QCD and discuss its relation to observables in diffractive leptoproduction of heavy quarkonium. The dipole cross section calculated with the unintegrated gluon density of the nucleon substantially differs from the well-known perturbative form $\sigma_{dip} \sim b^2$ for $b > 0.3$fm, where $b$ is the transverse separation of the dipole. We show the measured ratio of $\psi'$ to $J/\psi$ photoproduction cross sections constrains the dipole cross section at intermediate $b$, and in fact excludes the simple $\sigma_{dip} \sim b^2$ behavior. We also calculate the $t$-slopes of the diffractive $J/\psi, \psi'$ productions. We emphasize the difference of $t$-slopes, $B_{J/\psi} - B_{\psi'}$, is dominated by the dipole-nucleon dynamics. This difference is found to be about 0.3GeV$^{-2}$ with our dipole cross section.

## 1. Introduction

Recently hard diffractive processes in QCD have attracted considerable theoretical and experimental interests. In particular, diffractive photo- and leptoproductions of vector mesons off the proton, $\gamma^{(*)} + p \to V(\rho^0, \omega, \phi, J/\psi, \ldots) + p$, provide crucial constraints on gluon dynamics at small-$x$[1–5]. In the target rest frame, this process can be described by the color dipole picture, where the incoming photon undergoes $q\bar{q}$ fluctuations and these components subsequently interact with the target nucleon[6]. Within this picture, a number of exclusive and inclusive processes can be formulated in terms of a *universal* color dipole-nucleon cross section by virtue of the QCD factorization theorem.

Behavior of the dipole-nucleon cross section $\sigma_{dip}(b)$ is of great interest, where $b$ is a transverse size of the dipole, especially for intermediate region ($b \sim$ 0.3-0.4fm). There are several phenomenological works to determine the dipole cross section to reproduce the experimental data[7]. In this work, we calculate the dipole-nucleon cross section within perturbative QCD. Hence, we deal with the process which contains a hard scale such as a large momentum transfer $Q^2, t$ or the heavy quark mass. Since we are interesetd in the vector meson production at HERA energy, typically $x \sim 10^{-3}$, we do not cosider the saturation of the gluon density at small-$x$.[3] We discuss how resulting dipole cross section differs from the well-known perturbative $\sigma_{dip} \sim b^2$ behavior, and calculate observables which are sensitive to the behavior of the dipole cross section.

## 2. Hard color dipole cross section

Leading order diagrams for dipole-nucleon scattering, $q\bar{q}(q) + p(p) \to q\bar{q}(q+\Delta) + p(p-\Delta)$, are shown in Fig.1. With the unintegrated gluon density of the proton $f(x, l_T)$, the dipole cross section at forward limit, $t = \Delta^2 = 0$, is expressed as

$$\sigma_{dip} = \frac{4\pi^2\alpha_s}{3} \int^{Q^2} dl_T^2 \; [1 - J_0(b\, l_T)] \, \frac{f(x, l_T^2)}{l_T^4} \quad (1)$$

where $Q^2$ is the virtuality of the incoming photon which sets the scale of the process, and $x, l_T$ are

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[3]If we focus on much smaller-$x$, e.g. $x \sim 10^{-5}$, we should include the saturation effects on the dipole-nucleon cross section.

longitudinal momentum fraction and transverse momentum of the gluon, respectively. Our calculation is essentially based on the $k_T$-factorization scheme. Relation between $Q^2$ and the transverse distance $b$ is studied in ref. [8].

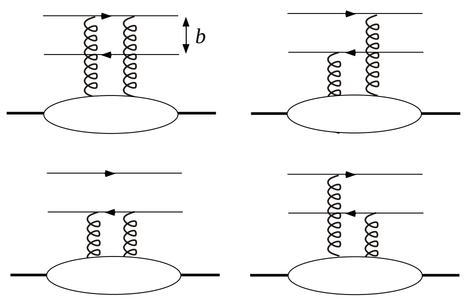


Figure 1. Diagrams for dipole-nucleon scattering

The unintegrated gluon density is not precisely known at present. In principle, there is a $Q^2$ dependence in $f(x, l_T^2, Q^2)$, and such a scale dependence can be studied, say, by solving CCFM equation[9], although we do not incorporate it in this work. Here, we simply assume that the unintegrated gluon density is related to the conventional gluon distribution measured in deep inelastic scattering via

$$f(x, l_T) \equiv l_T^2 \frac{\partial\, xG(x, l_T^2)}{\partial\, l_T^2} \ . \tag{2}$$

Before going to evaluate eq. (1) explicitly, we shall show an approximate expression for the small size dipole. Assuming $bl_T << 1$ and keeping terms up to second order of $(b \cdot l_T)$, we rewrite eq. (1) as

$$\sigma_{dip} = \frac{\pi^2 \alpha_s}{3}\ b^2\, xG(x, Q^2) \tag{3}$$

We refer this approximation as the small dipole approximation (SDA), in which the dipole cross section has a simple geometrical expression $\sigma_{dip} \sim b^2$. This expression is frequently used for various applications[6]. It is clear that this approximation works well for small-$b$ region, or, in other words, the process with large $Q^2$.

Now we come back to eq. (1). To integrate out eq. (1), we use the parametrization of the gluon distribution function. However, available parametrizations are known for $Q^2 \geq$ a few GeV$^2$. Here, we follow one of the prescriptions made in ref. [10]: the linear extrapolation for $Q^2$ below $Q_0^2$,

$$G(x, Q^2) = \begin{cases} G(x, Q^2) & Q^2 \geq Q_0^2 \\ \frac{Q^2}{Q_0^2} G(x, Q_0^2) & Q^2 \leq Q_0^2 \end{cases} \tag{4}$$

where $Q_0^2$ is an initial input scale of the parametrizations $\sim 1\mathrm{GeV}^2$.

Calculated dipole cross section is shown in Fig.2 at $x = 10^{-3}$. Solid curve denotes our full result eq. (1), while the dashed one indicates the result with SDA (3). Both results agree at small $b < 0.2$fm as expected. However, difference between them is substantial at $b = 0.3 \sim 0.4$fm. Since we use the perturbative technique, the result at $b \sim 1$fm cannot be justified. Nevertheless, there is a distinct deviation at intermediate $b$, which is crucial for observables discussed in the following sections. To get the full result of Fig.2, we used CTEQ5m[11] with the linear extrapolation. We have also used several sets of parametrization of the gluon distribution and other approximation scheme for the low $Q^2$ region. The difference is about $10 \sim 20\%$.

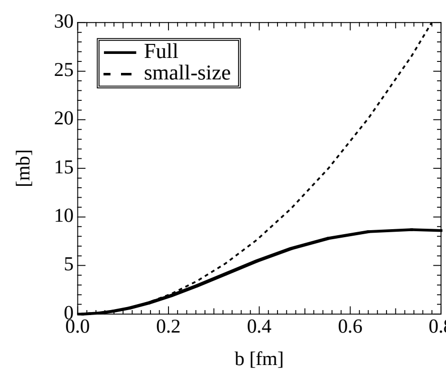


Figure 2. Color dipole-nucleon cross section

## 3. $\psi'$ to $J/\psi$ ratio

Using the dipole cross section, one can write the diffractive vector meson production amplitude symbolically as;

$$\mathcal{A} = \psi_\gamma \otimes \sigma_{dip} \otimes \psi_V \tag{5}$$

where $\psi_\gamma, \psi_V$ are light-cone wave functions of the photon and vector meson, respectively. In eq. (5) integrations are performed over $z$, longitudinal momentum fraction of a heavy quark, and $b$. Typical scale of this process is $Q^2_{eff} = Q^2/4 + m^2$ where $m$ is the heavy quark mass.

The SDA approach is adopted to evaluate the $J/\psi$ production in refs. [1,3], and gives reasonable results in agreement with the data. However, as pointed out by Hoyer and Peigné[12] very recently, the ratio of $\psi'$ to $J/\psi$ photoproduction cross sections calculated in SDA is much smaller than the measured values[13], when one uses realistic non-relativistic quark models to obtain the charmonium wave functions. The formula used in SDA is obtained by assuming that $(b \cdot l_T)$ is small as discussed in section 2. However, the quark model indicates that the size of $\psi' \sim 0.8$fm is twice as large as the radius of $J/\psi$. Hence, the validity of SDA is questionable for $\psi'(2S)$ case. We may expect considerable contributions from the overlap of the large size color dipole and $\psi'$ wave function. Above argument motivates us to calculate the diffractive $J/\psi$ and $\psi'$ productions without resorting to the small dipole approximation[14].

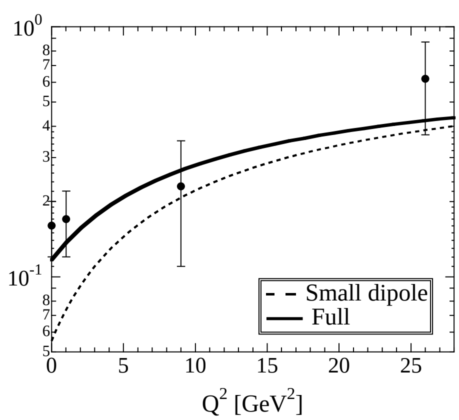


Figure 3. $\psi'$ to $J/\psi$ production ratio

We obtain the results shown in Fig.3 with the dipole cross section of Fig.2[14]. Calculation without (with) the small dipole approximation is depicted by the solid (dashed) curve. For large $Q^2$ the difference between them becomes small as expected. However, around $Q^2 = 0$, the full calculation with eqs. (1) and (2) is clearly larger than the result obtained by eq. (3) by factor about 2, and shows a reasonable agreement with the experimental data[13].

We also consider the production of $\Upsilon$-states. Results for $\Upsilon'(2S)/\Upsilon(1S)$ and $\Upsilon'(3S)/\Upsilon(1S)$ are consistent with those obtained by SDA. SDA is already enough for the $\Upsilon$ production due to the large bottom quark mass.

## 4. $t$-slope of heavy quarkonium production

Measured diffractive production cross section of the vector meson at small momentum transfer, $t \neq 0$, can be parametrized as,

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} \cdot \exp(Bt) \qquad (6)$$

where the diffractive slope $B$ is a function of $W, Q^2, t$ in general. At low $t$, we may decompose the $t$-slope $B$ as

$$B = B_{dip} + B_{Soft} + \cdots \ , \qquad (7)$$

where $B_{dip}$ indicates contributions from the color dipole-nucleon dynamics, and the remainder expresses other possible contributions including the soft nucleon structure $B_{Soft}$.

The contribution from the dipole part for small $t$ is given by[3]

$$B_{dip} \propto \psi_\gamma \otimes b^2 \sigma_{dip} \otimes \psi_V \ . \qquad (8)$$

Our results for $B_{dip}$ are shown in Fig.4 for $J/\psi, \psi'$, although $B_{dip}$ cannot be directly compared with the data. Here, $L$ and $T$ denote the longitudinal and transverse polarizations of the photon. For $J/\psi$, results with the full dipole cross section and SDA give almost the same behavior. However, for $\psi'$, there is a clear difference. Such large differences come from the intermediate-$b$ behavior of the dipole cross section. In turn, the $t$-slope of the $\psi'$ production could be useful to constrain the behavior of the dipole-nucleon cross section for $b > 0.3$fm.

Let us consider the difference of $t$-slopes, $B_{J/\psi} - B_{\psi'}$. Because the contribution from soft

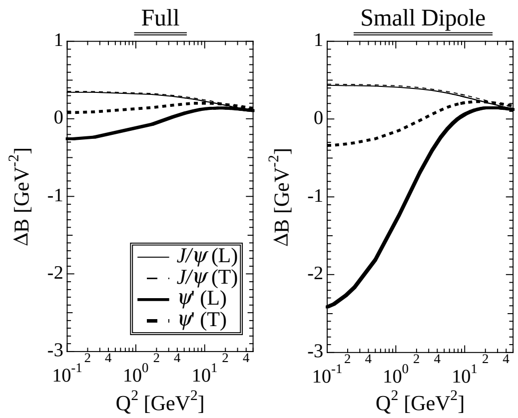


Figure 4. $B_{nS}^{dip}$ for various photon polarizations

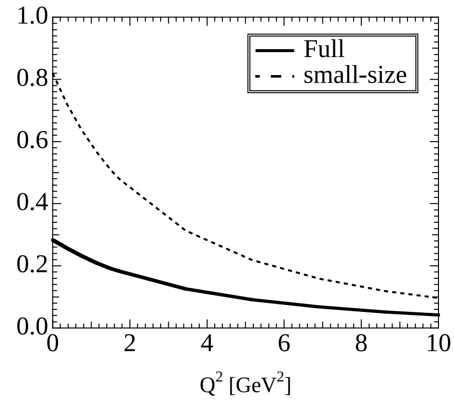


Figure 5. $B_{J/\psi} - B_{\psi'}$ [GeV$^{-2}$]

nucleon structure in eq. (7) is independent of wave functions of the vector mesons, it must be the same for $J/\psi$ and $\psi'$ productions. Therefore, it is easy to arrive at a relation;

$$B_{J/\psi} - B_{\psi'} = B_{dip}(J/\psi) - B_{dip}(\psi') \qquad (9)$$

which suggests $B_{J/\psi} - B_{\psi'}$ is dominated by the dipole contribution, and thus a suitable quantity to study the shape of the dipole-nucleon cross section. We calculate this difference for the transversely polarized case shown in Fig.5. Results with the full dipole cross section gives a considerably smaller value than that of SDA. Recent preliminary data from HERA[15] is rather consistent with our full calculation. More accurate data is quite important for detailed studies.

## 5. Conclusion

We have calculated the $q\bar{q}$ dipole-nucleon cross section in terms of the unintegrated gluon density. The dipole cross section is the universal quantity which characterizes the various hard processes. We have found the $\psi'$ to $J/\psi$ diffractive production cross section ratio is sensitive to the behavior of $\sigma_{dip}$ for $b > 0.3$fm. We have demonstrated the major improvement of the ratio is caused by the use of realistic dipole cross section. We have also discussed $t$-slope of the diffractive leptoproduction of $J/\psi$ and $\psi'$. We have emphasized that the difference of $B_{J/\psi} - B_{\psi'}$ is sensitive to the shape of the dipole cross section. Our result for the photoproduction is about 0.3GeV$^{-2}$, which seems to be consistent to recent preliminary data.

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